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Review Article

DIMENSIONLESS REYNOLDS NUMBER AS A DIMENSION FOR FLUID MECHANICS IN RHEOLOGY

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ABSTRACT

Reynolds number, in fluid mechanics, a criterion of whether fluid (liquid or gas) flow is absolutely steady (streamlined, or laminar) or on the average steady with small unsteady fluctuations (turbulent). Whenever the Reynolds number is less than about 2,000, flow in a pipe is generally laminar, whereas, at values greater than 2,000, flow is usually turbulent. Actually, the transition between laminar and turbulent flow occurs not at a specific value of the Reynolds number but in a range usually beginning between 1,000 to 2,000 and extending upward to between 3,000 and 5,000.

In 1883 Osborne Reynolds, a British engineer and physicist, demonstrated that the transition from laminar to turbulent flow in a pipe depends upon the value of a mathematical quantity equal to the average velocity of flow times the diameter of the tube times the mass density of the fluid divided by its absolute viscosity. This mathematical quantity, a pure number without dimensions, became known as the Reynolds number and was subsequently applied to other types of flow that are completely enclosed or that involve a moving object completely immersed in a fluid.

Keywords: Newtonian Flow, Non Newtonian Flow, Bingham Flow, Laminar Flow, Turbulent Flow, Plastic Flow, Pseudo plastic Flow, Dilatant Flow, Thixotropic Flow, Transitional Flow

INTRODUCTION

Rheology is the study of flow and deformation of materials under applied forces which is routinely measured using a rheometer. The measurement of rheological properties is applicable to all materials – from fluids such as dilute solutions of polymers and surfactants through to concentrated protein formulations, to semi-solids such as pastes and creams, to molten or solid polymers as well as asphalt. Rheological properties can be measured from bulk sample deformation using a mechanical rheometer or on a micro-scale by using a micro capillary viscometer or an optical technique such as Micro-rheology. Many commonly-used materials and formulations exhibit complex rheological properties, whose viscosity and visco-elasticity can vary depending upon the external conditions applied, such as stress, strain, timescale and temperature. Internal sample variations such as protein concentration and stability and formulation type for biopharmaceuticals are also key factors that determine rheological properties.[1]

Rheological properties impact at all stages of material use across multiple industries – from formulation development and stability to processing and product performance. The type of rheometer required for measuring these properties is often dependent on the relevant shear rates and timescales as well as sample size and viscosity. Examples of rheological measurements include: 1. Viscosity profiling for non-Newtonian shear-dependent behaviour to simulate processing or in-use conditions. 2. Viscoelastic fingerprinting for material classification to determine extent of solid-like or liquid-like behavior. 3. Optimizing and assessing dispersion stability. 4. Determination of thixotropy of paints and coatings for product application and final finish quality. 5. Impact of molecular architecture of polymers on visco-elasticity for processing and end-use performance. 6. Benchmarking Food and Personal Care products for ability to pump or spread. 7. Full cure profiling for bonding or gelling systems. 8. Pre-formulation screening for therapeutics, particularly biopharmaceuticals.

Laminar Flow is the flow of a fluid when each particle of the fluid follows a smooth path, paths which never interfere with one another. One result of laminar flow is that the velocity of the fluid is constant at any point in the fluid. Turbulent Flow is irregular flow that is characterized by tiny whirlpool regions. Turbulent flow, type of fluid (gas or liquid) flow in which the fluid undergoes irregular fluctuations, or mixing, in contrast to laminar flow, in which the fluid moves in smooth paths or layers. In turbulent flow the speed of the fluid at a point is continuously undergoing changes in both magnitude and direction. The flow of wind and rivers is generally turbulent in this sense, even if the currents are gentle. The air or water swirls and eddies while its overall bulk moves along a specific direction. Most kinds of fluid flow

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are turbulent, except for laminar flow at the leading edge of solids moving relative to fluids or extremely close to solid surfaces, such as the inside wall of a pipe, or in cases of fluids of high viscosity (relatively great sluggishness) flowing slowly through small channels. Common examples of turbulent flow are blood flow in arteries, oil transport in pipelines, lava flow, atmosphere and ocean currents, the flow through pumps and turbines and the flow in boat wakes and around aircraft-wing tips.

Reynolds number is used to check whether the flow is laminar or turbulent. It is denoted by R_e. This number got by comparing inertial force with viscous force. Reynolds Number=Inertial force/Viscous force. L is the length or diameter of the fluid. Reynolds number formula is used in the problems to find the velocity (V), density (ρ), viscosity (μ) and diameter (L) of the fluid. It is dimensionless. If $2000<Re<4000$ it is called transition flow.[2]

The Reynolds number (R_e) is an important dimensionless quantity in fluid mechanics that is used to help predict flow patterns in different fluid flow situations. It is widely used in many applications ranging from liquid flow in a pipe to the passage of air over an aircraft wing. The Reynolds Number is valuable as a guide to the laminar-turbulent transition in a particular flow situation and for the scaling of similar but different-sized flow situations, such as between an aircraft model in a wind tunnel and the full size version. The predictions of onset of turbulence and the ability to calculate scaling effects can be used to help predict fluid behaviour on a larger scale, such as in local or global air or water movement and thereby the associated meteorological and climatological effects. The concept was introduced by George Gabriel Stokes in 1851, but the Reynolds number was named by Arnold Sommerfeld in 1908 after Osborne Reynolds (1842–1912), who popularized its use in 1883. Reynolds Number=Inertial force/Viscous force; $R_e=\rho VL/\mu$.

Where, (ρ) rho is the density of the fluid, (V) is the velocity of the fluid, (μ) mu is the viscosity of fluid, (L) is the length or diameter of the fluid. Reynolds number formula is used in the problems to find the Velocity (V), density (ρ), Viscosity (μ) and diameter (L) of the fluid. The Kind of flow depends on value of R_e; 1. If $Re<2000$ the flow is Laminar. 2. If $Re>4000$ the flow is turbulent. 3. If $2000<Re<4000$ it is called transition flow.[3]

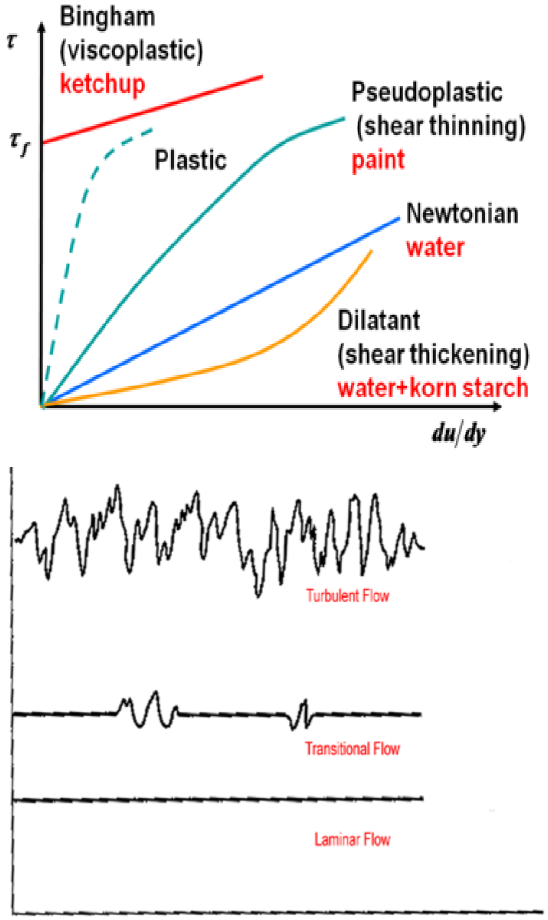

Figure 1: Fluid flow

Definition

The Reynolds number is the ratio of inertial forces to viscous forces within a fluid which is subjected to relative internal movement due to different fluid velocities, in what is known as a boundary layer in the case of a bounding surface such as the interior of a pipe. A similar effect is created by the introduction of a stream of higher velocity fluid, such as the hot gases from a flame in air. This relative movement generates fluid friction, which is a factor in developing turbulent flow. Counteracting this effect is the viscosity of the fluid, which as it increases, progressively inhibits turbulence, as more kinetic energy is absorbed by a more viscous fluid. The Reynolds number quantifies the relative importance of these two types of forces for given flow conditions and is a guide to when turbulent flow will occur in a particular situation.

This ability to predict the onset of turbulent flow is an important design tool for equipment such as piping systems or aircraft wings, but the Reynolds number is also used in scaling of fluid dynamics problems and is used to determine dynamic similitude between two different cases of fluid flow, such as between a model aircraft and its full size version. Such scaling is not linear and the application of Reynolds numbers to both situations allows scaling factors to be developed.

With respect to laminar and turbulent flow regimes:

- Laminar flow occurs at low Reynolds numbers, where viscous forces are dominant and is characterized by smooth, constant fluid motion;

- Turbulent flow occurs at high Reynolds numbers and is dominated by inertial forces, which tend to produce chaotic eddies, vortices and other flow instabilities.

The Reynolds number is defined as
where:

- ρ is the density of the fluid (SI units: kg/m^3)
- V is a characteristic velocity of the fluid with respect to the object (m/s)
- L is a characteristic linear dimension (m)
- μ is the dynamic viscosity of the fluid (Pa·s or N·s/m^2 or kg/(m·s)).

The Reynolds number can be defined for several different situations where a fluid is in relative motion to a surface. These definitions generally include the fluid properties of density and viscosity, plus a velocity and a characteristic length or characteristic dimension (L in the above equation). This dimension is a matter of convention – for example radius and diameter are equally valid to describe spheres or circles, but one is chosen by convention. For aircraft or ships, the length or width can be used. For flow in a pipe or a sphere moving in a fluid the internal diameter is generally used today. Other shapes such as rectangular pipes or non-spherical objects have an equivalent diameter defined. For fluids of variable density such as compressible gases or fluids of variable viscosity such as non-Newtonian fluids, special rules apply. The velocity may also be a matter of convention in some circumstances, notably stirred vessels.[4]

In practice, matching the Reynolds number is not on its own sufficient to guarantee similitude. Fluid flow is generally chaotic and very small changes to shape and surface roughness can result in very different flows. Nevertheless, Reynolds numbers are a very important guide and are widely used.

Derivation

The form of the Reynolds number can be derived as follows:
R_e=Inertialforces/Viscous forces=(mass)×(acceleration)/(dynamic viscosity)×(velocity/distance)×(area)
$=(\rho L^3)\times[(V/t)/\mu]\times(V/L)\times L^2=(\rho L^3)\times[(1/t)/\mu]\times(1/L)\times L^2=(\rho L^2)\times(1/t)/\mu=(\rho)\times(L/t)\times(L)/\mu=\rho VL/\mu=VL/\nu$

where:

- V is the maximum velocity of the object relative to the fluid (SI units: m/s)
- L is a characteristic linear dimension, (travelled length of the fluid; hydraulic diameter when dealing with river systems) (m)
- t denotes the time
- μ is the dynamic viscosity of the fluid (Pa·s or N·s/m^2 or kg/(m.s))
- ν (nu) is the kinematic viscosity ($\nu=\mu/\rho$) (m^2/s)
- ρ is the density of the fluid (kg/m^3).

Note that multiplying the Reynolds number by Lv/Lv yields $\rho v^2 L^2/\mu v L$, which is the ratio of the inertial forces to the viscous forces. It could also be considered the ratio of the total momentum transfer to the molecular momentum transfer.

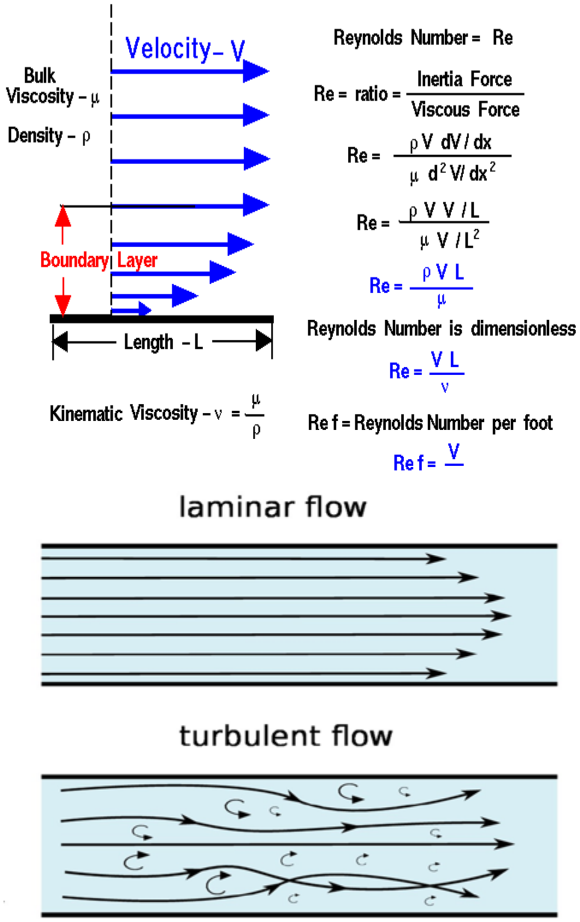

Figure 2: Rheology

Flow in pipe

For flow in a pipe or tube, the Reynolds number is generally defined as:
where:
$Re=\rho VD_H/\mu=VD_H/\nu=QD_H/\nu A$

- D_H is the hydraulic diameter of the pipe (the inside diameter if the pipe is circular) (m).
- Q is the volumetric flow rate (m^3/s).
- A is the pipe's cross-sectional area (m^2).
- V is the mean velocity of the fluid (m/s).
- μ is the dynamic viscosity of the fluid ($Pa{\cdot}s=N{\cdot}s/m^2=kg/(m*s)$).
- ν (nu) is the kinematic viscosity ($\nu=\mu/\rho$) (m^2/s).
- ρ is the density of the fluid (kg/m^3).

For shapes such as squares, rectangular or annular ducts where the height and width are comparable, the characteristically dimension for internal flow situations is taken to be the hydraulic diameter, D_H, defined as: $D_H=4A/P$; where A is the cross-sectional area and P is the wetted perimeter. The wetted perimeter for a channel is the total perimeter of all channel walls that are in contact with the flow. This means the length of the channel exposed to air is not included in the wetted perimeter.

For a circular pipe, the hydraulic diameter is exactly equal to the inside pipe diameter, D. That is, $D_H=D$

For an annular duct, such as the outer channel in a tube-in-tube heat exchanger, the hydraulic diameter can shown algebraically to reduce to $D_{H,\ annulus}=D_o-D_i$

where,

D_o is the inside diameter of the outside pipe, and D_i is the outside diameter of the inside pipe.

For calculations involving flow in non-circular ducts, the hydraulic diameter can be substituted for the diameter of a circular duct, with reasonable accuracy, if the aspect ratio AR of the duct cross-section remains in the range $1/4<AR<4$.[5]

Laminar-turbulent transition

In boundary layer flow over a flat plate, experiments confirm that, after a certain length of flow, a laminar boundary layer will become unstable and turbulent. This instability occurs across different scales and with different fluids, usually when $R_{ex}\approx5\times10^5$, where x is the distance from the leading edge of the flat plate and the flow velocity is the free stream velocity of the fluid outside the boundary layer. For flow in a pipe of diameter D, experimental observations show that for fully developed flow, laminar flow occurs when $R_{eD}<2300$ and turbulent flow occurs when $R_{eD}>4000$. In the interval between 2300 and 4000, laminar and turbulent flows are possible and are called transition flows, depending on other factors, such as pipe roughness and flow uniformity. This result is generalized to non-circular channels using the hydraulic diameter, allowing a transition Reynolds number to be calculated for other shapes of channel. These transitions Reynolds numbers are also called critical Reynolds numbers and were studied by Osborne Reynolds around 1895. The critical Reynolds number is different for every geometry.[6]

Flow in wide duct

For a fluid moving between two plane parallel surfaces—where the width is much greater than the space between the plates—then the characteristic dimension is equal to the distance between the plates.

Flow in open channel

For flow of liquid with a free surface, the hydraulic radius must be determined. This is the cross-sectional area of the channel divided by the wetted perimeter. For a semi-circular channel, it is half the radius. For a rectangular channel, the hydraulic radius is the cross-sectional area divided by the wetted perimeter. Some texts then use a characteristic dimension that is four times the hydraulic radius, chosen because it gives the same value of R_e for the onset of turbulence as in pipe flow, while others use the hydraulic radius as the characteristic length-scale with consequently different values of R_e for transition and turbulent flow.

Flow around airfoils

Reynolds numbers are used in airfoil design to (among other things) manage Scale Effect when computing/comparing characteristics (a tiny wing, scaled to be huge, will perform differently). Fluid dynamicists define the chord Reynolds number, R, like this: $R=Vc/\nu$ where V is the flight speed, c is the chord length and ν is the kinematic viscosity of the fluid in which the airfoil operates, which is $1.460\times10^{-5}\ m^2/s$ for the atmosphere at sea level. In some special studies a characteristic length other than chord may be used; rare is the span Reynolds number which is not to be confused with span wise stations on a wing where chord is still used.[7]

Object in fluid

The Reynolds number for an object in a fluid, called the particle Reynolds number and often denoted R_{ep}, is important when considering the nature of the surrounding flow, whether or not vortex shedding will occur and its fall velocity. Where the viscosity is naturally high, such as polymer solutions and polymer melts, flow is normally laminar. The Reynolds number is very small and Stoke's law can be used to measure the viscosity of the fluid. Spheres are allowed to fall through the fluid and they reach the terminal velocity quickly, from which the viscosity can be determined. The laminar flow of polymer solutions is exploited by animals such as fish and dolphins, which exude viscous solutions from their skin to aid flow over their bodies while swimming. It has been used in yacht racing by owners who want to gain a speed advantage by pumping a polymer solution such as low molecular weight polyoxyethylene in water, over the wetted surface of the hull. It is, however, a problem for mixing of polymers, because turbulence is needed to distribute fine filler through the material. Inventions such as the cavity transfer mixer have been developed to produce multiple folds into a moving melt so as to improve mixing efficiency. The device can be fitted onto extruders to aid mixing.[8]

Typical values of Reynolds number: Ciliate~1×10^{-1}; Smallest fish~1; Blood flow in brain~1×10^{2}; Blood flow in aorta~1×10^{3}.

Onset of turbulent flow~2.3×10^{3} to 5.0×10^{4} for pipe flow to 10^{6} for boundary layers: Typical pitch in major league baseball~2×10^{5}; Person swimming~4×10^{6}; Fastest fish~1×10^{8}; Blue whale~4×10^{8}; A large ship~5×10^{9}.

Conclusion

In fluid dynamics, laminar flow (or streamline flow) occurs when a fluid flows in parallel layers, with no disruption between the layers. At low velocities, the fluid tends to flow without lateral mixing and adjacent layers slide past one another like playing cards. There are no cross-currents perpendicular to the direction of flow, nor eddies or swirls of fluids. In laminar flow, the motion of the particles of the fluid is very orderly with particles close to a solid surface moving in straight lines parallel to that surface. Laminar flow is a flow regime characterized by high momentum diffusion and low momentum convection. When a fluid is flowing through a closed channel such as a pipe or between two flat plates, either of two types of flow may occur depending on the velocity and viscosity of the fluid: laminar flow or turbulent flow. Laminar flow tends to occur at lower velocities, below a threshold at which it becomes turbulent. Turbulent flow is a less orderly flow regime that is characterised by eddies or small packets of fluid particles which result in lateral mixing. In non-scientific terms, laminar flow is smooth while turbulent flow is rough. Rheology is the study of the flow of matter, primarily in a liquid state, but also as soft solids or solids under conditions in which they respond with plastic flow rather than deforming elastically in response to an applied force. The term was first used by Eugene C. Bingham and Crawford to describe the flow of liquids and the deformation of solids. It applies to substances which have a complex microstructure, such as muds, sludges, suspensions, polymers and other glass formers (e.g., silicates), as well as many foods and additives, body fluids (e.g., blood) and other biological materials or other materials which belong to the class of soft matter such as food.

Newtonian fluids can be characterized by a single coefficient of viscosity for a specific temperature. Although this viscosity will change with temperature, it does not change with the strain rate. Only a small group of fluid exhibit such constant viscosity. The large class of fluids whose viscosity changes with the strain rate (the relative flow velocity) are called non-Newtonian fluids. Rheology generally accounts for the behaviour of non-Newtonian fluids, by characterizing the minimum number of functions that are needed to relate stresses with rate of change of strain or strain rates. For example, ketchup can have its viscosity reduced by shaking (or other forms of mechanical agitation, where the relative movement of different layers in the material actually causes the reduction in viscosity) but water cannot. Ketchup is a shear thinning material, like yogurt and emulsion paint (US terminology latex paint or acrylic paint), exhibiting thixotropy, where an increase in relative flow velocity will cause a reduction in viscosity, for example, by stirring. Some other non-Newtonian materials show the opposite behaviour, rheopecty: viscosity going up with relative deformation and are called shear thickening or dilatant materials. Since Sir Isaac Newton originated the concept of viscosity, the study of liquids with strain rate dependent viscosity is also often called Non-Newtonian fluid mechanics. The term rheology was coined by Eugene C. Bingham, a professor at Lafayette College, in 1920, from a suggestion by a colleague, Markus Reiner. The term was inspired by the aphorism of Simplicius (often attributed to Heraclitus), everything flows. The experimental characterization of a material's rheological behaviour is known as rheometry, although the term rheology is frequently used synonymously with rheometry, particularly by experimentalists. Theoretical aspects of rheology are the relation of the flow/deformation behaviour of material and its internal structure (e.g., the orientation and elongation of polymer molecules) and the flow/deformation behaviour of materials that cannot be described by classical fluid mechanics or elasticity. In fluid mechanics, the Reynolds number is a measure of the ratio of inertial forces ($v_s\rho$) to viscous forces (μ/L) and consequently it quantifies the relative importance of these two types of effect for given flow conditions. Under low Reynolds numbers viscous effects dominate and the flow is laminar, whereas at high Reynolds numbers inertia predominates and the flow may be turbulent. However, since rheology is concerned with fluids which do not have a fixed viscosity, but one which can vary with flow and time, calculation of the Reynolds number can be complicated. It is one of the most important dimensionless numbers in fluid dynamics and is used, usually along with other dimensionless numbers, to provide a criterion for determining dynamic similitude. When two geometrically similar flow patterns, in perhaps different fluids with possibly different flow rates, have the same values for the relevant dimensionless numbers, they are said to be dynamically similar.

Typically it is given as follows:

Where: mean flow velocity, [m s^{-1}], characteristic length, [m], (absolute) dynamic fluid viscosity, [N s m^{-2}] or [Pa s], kinematic fluid viscosity: $\nu=\mu/\rho$, [m^2 s^{-1}], fluid density, [kg m^{-3}].

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